Validated Engineering Model for Residual Strength Prediction

Deliverable No. 25

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Version 1
Confidential

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<th>pages</th>
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<td>14-6-2006</td>
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</tbody>
</table>
1 SUMMARY ...............................................................................................................................5
2 INTRODUCTION .........................................................................................................................6
3 THEORETICAL MODELS .............................................................................................................8
4 RESIDUAL TENSILE STRENGTH .................................................................................................12
   4.1 UD at 0° ................................................................................................................................12
      4.1.1 R=0.1 ............................................................................................................................12
      4.1.2 R=-1 ...........................................................................................................................14
   4.2 UD at 90° ................................................................................................................................17
      4.2.1 R=0.1 ............................................................................................................................17
      4.2.2 R=-1 ...........................................................................................................................18
      4.2.3 R=10 ............................................................................................................................20
   4.3 SHEAR ....................................................................................................................................22
      4.3.1 R=0.1 ............................................................................................................................22
5 RESIDUAL COMPRESSIVE STRENGTH .......................................................................................25
6 VALIDATION OF PROPOSED MODELS ....................................................................................29
   6.1 MD DEGRADATION CURVES ...............................................................................................29
   6.2 STATISTICAL PREDICTIONS ...............................................................................................32
7 CONCLUSIONS ..........................................................................................................................36
8 REFERENCES ..............................................................................................................................38
1 SUMMARY

This work was performed in the frame of WP13: ‘Residual Strength and Condition Assessment’, TG5, of OPTIMAT BLADES. The objective was to propose engineering models for the prediction of residual strength degradation due to fatigue, observed on the various OB Glass Epoxy laminates. It is the main deliverable (#25) of task 13.4 of TG5, entitled “Establishment of predictive engineering model”. It constitutes the continuation of report OB_TG5_R003, deliverable #6, entitled “Review of existing residual strength predictive models” [1]. Specific engineering residual strength models, applicable for prediction of residual strength degradation on the OB laminates investigated, are defined explicitly. The models proposed are implemented on experimental data from tests on OB_UD and ISO ±45 coupons, generated in the frame of WP13, task 13.2: “Experimental evaluation of residual strength after fatigue” of TG5. These models are further on used for estimating the residual strength behaviour of MD laminates, under similar loading conditions, while the theoretical predictions are validated by comparing with experimental data.
2 INTRODUCTION

Static strength degradation due to fatigue of materials used in wind turbine rotor blades has been studied extensively in the frame of OPTIMAT BLADES. For the first time, unidirectional and multidirectional laminates have been tested for their residual strength both in tension and in compression, after being fatigued at a variety of stress ratios, stress levels and fractions of their nominal lives once the static and fatigue behaviour were defined. The results indicate interesting dependencies between residual strength, stress level and loading direction. Validated engineering residual strength models, as presented in this report, should provide the knowledge for choosing between different modelling methodologies and characterization procedures requiring reasonable experimental effort. Such methodologies combined with stiffness degradation models and implemented to numerical codes simulating at the lamina level of a composite laminate, fatigued under complex stress states, is an alternative for modelling damage accumulating into the laminate and predicting incipient and catastrophic failure.

Residual strength tests performed during this project consist of static tests up to fracture in tension (RST) and in compression (RSC) after cyclic loading of the specimens. During cycling, three stress ratios are used; tension-tension (R=0.1), reverse loading (R=-1) and compression-compression (R=10). Three stress level per S-N curve and three life fractions per stress level are chosen, as described in the DPA of TG5 [2].

Various residual strength models are presented in report OB_TG5_R003 “Review of residual strength models” [1]. They all are phenomenological models assessed on their simplicity and accurate description of the phenomenon while requiring as limited an experimental effort as possible. Predictions of the probability distribution of residual strength are also considered. Unfortunately, variations between different test rigs, slightly different test conditions and finally property variations of the coupons produced by LM in the course of the project affect drastically the validity of data statistical evaluation. One consequence of that is that prematurely failed coupons cannot be used for validation of the statistical models, and are disregarded.

The variation of material properties from plate to plate, due to manufacturing conditions, is probably the cause that certain coupons exhibited residual strength higher than the measured average initial static strength, especially at higher stress levels where less
degradation occurs. Although similar behaviour has been reported in the literature of composite materials, [3]-[14], it is mostly related to test results from notched coupons and is usually attributed to relaxation of stress concentrations in the vicinity of the notch. Consequently, it was considered safe not to develop more complex models that would account for such increase of strength after low cycle fatigue, but to neglect test results generated from coupons cut from a ‘suspect’ plate or presenting high residual static strength, beyond any statistical explanation.
3 THEORETICAL MODELS

From the various models discussed in [1], those finally proposed for the description of residual strength of the Glass fibre/Epoxy system, all different lay-ups, investigated in the OPTIMAT BLADES are presented below, in brief.

(a) The interaction model (INT) by Harris and associates [16] is based on the fitting of a two-parameter equation to a data set of normalized residual strength, \( r \), and normalized life, \( t \):

\[
\left( \frac{\log(n) - \log(0.5)}{\log(N) - \log(0.5)} \right)^x + \left( \frac{X - \sigma_{\text{max}}}{X - \sigma_{\text{max}}^\text{max}} \right)^y = 1, \text{ or } t^x + r^y = 1
\]  

(1)

The resulting degradation curve from eq.(1) is flexible enough to predict various degradation trends, from 'initial loss of strength' models to 'sudden death' ones.

No statistical prediction is proposed by the authors. A probabilistic expansion based on the distribution of static strength was derived to produce a CDF prediction as described in [1]. The procedure relies on the substitution of eq.(1) into the static strength distribution (Weibull). The resulting equation is:

\[
P_x(X_r \leq x / X > \sigma_{\text{max}}) = 1 - \exp \left[ - \left( \frac{x - \sigma_{\text{max}} + \sigma_{\text{max}} (1 - t^x)^\frac{1}{y}}{\beta (1 - t^x)^\frac{1}{y}} \right)^a + \left( \frac{\sigma_{\text{max}}}{\beta} \right)^a \right]
\]  

(2)

(b) The linear model (BR) proposed for the first time by Broutman & Sahu [17], is the simplest and most cost-effective predictive model requiring no experimental characterization of residual strength. It consists of an equation of the following form:

\[
X_r = X - (X - \sigma_{\text{max}}) \left( \frac{n}{N} \right)
\]  

(3)

It considers linear strength degradation to failure, which although not accurate always, lies in most cases on the safe side of the experimental data.

The CDF of residual strength is derived based on the process discussed above resulting in:
The model, despite its complexity, suffers in predicting possible loss of strength in the beginning of coupon life (steep strength degradation). The implementation procedure is more complex than in the previous cases, although the experimental effort for characterizing the material is comparable to the previous ones. As described in [1], the implementation procedure requires two data sets: A residual strength data set (RSDS) and a fatigue life data set (FLDS), both at various stress levels. The derivation of parameters is based on the equivalent static strength concept (ESS), according to which eq.(5) is solved for the static strength for each experimental datum (fatigue or residual strength). The resulting – parametric- distribution of ‘fictitious’ static strength is optimized in order to match with the experimentally obtained static strength distribution. During this procedure, the FLDS is used for determination of parameters \( \omega, K, b \) and the RSDS for determination of parameter \( c \).

Model Y2 can numerically predict the probability distribution of residual static strength following the procedure below:

1. Assigning a series of values for the probability \( P_{X_r}(X_r) \), the equivalent static strength \( X \) corresponding to this probability level is calculated by [1]:

\[
X = \left( \frac{\sigma_{\text{max}}^c + Ks^b_n}{\beta^c} \right)^{1/\alpha} \ln \left( 1 - P_{X_r}(X_r) \right) 
\]  

(6)
2. The residual strength corresponding to each probability value is calculated from eq.(5) by substituting the $X$–value derived in step 1.

3. The sets of $P_{X_i}(x_i)$ and $X_i$ are plotted to produce the CDF of residual strength.

(d) The following one-parameter equation (REI) was used by Reifsnider et al. [19] to describe strength degradation:

$$X_i = X - (X - \sigma_{\text{max}}) \left( \frac{n}{N} \right)^k$$

(7)

Experimental results indicate that the exponent $k$ in most cases has not a constant value but is a function of life fraction and/or stress level. Several tests at a single stress level should be performed if direct fitting is used to define the parameter. Less tests are required if the equivalent static strength concept (ESS) [1] is adopted. For the data sets analyzed herein, the two methods produce similar results.

Apparently, the REI model reduces to the linear one (BR) for $k=1$. The power law eq.(7) is convenient for modelling the ‘sudden death’ behaviour, i.e. constant strength throughout the coupon’s life and sudden degradation prior to failure. Such a behaviour is observed e.g. at the residual compressive strength tests, and is modelled by setting a high value to the exponent $k$. This special case is called SD model.

The prediction of the CDF of residual strength is derived using the same concept as in (a) leading to:

$$P(X_i \leq x / X > \sigma_{\text{max}}) = 1 - \exp \left\{ \frac{\sigma_{\text{max}}}{\beta} \left[ \frac{\sigma_{\text{max}}}{n N} \right]^k \right\}$$

$$- \exp \left\{ \frac{x - \sigma_{\text{max}}}{\beta} \left[ 1 - \left( \frac{n}{N} \right)^k \right] \right\}$$

(8)

(e) OM model. Considering the dependency of the power law model to cyclic load characteristics, various formulations of the exponent $k$ as functions of simple parameters,
such as maximum stress or life fraction have been investigated. In some cases, e.g. for the residual shear strength, there is a clear, non-linear dependency of $k$ on life fraction of the form:

$$k = k_1 \exp\left( \frac{n}{N} \right)$$  \hspace{1cm} (9)
4 RESIDUAL TENSILE STRENGTH

4.1 UD at 0°

4.1.1 R=0.1

Tests were performed in CCLRC, VUB and WMC. The residual strength tests at this stress ratio indicate a relatively large number of premature failures. This could be attributed mainly to test conditions since plate numbers of the coupons that failed prematurely vary from 42 to 177, even though such failures appear to be more frequent at higher plate numbers. Especially for the tests performed at VUB, an inexplicably high number of premature failures occurred even before 20% of nominal life, possibly due to the plate the coupons originated from (plate 113).

For the coupons that reached the target life fraction and were tested statically afterwards, a considerable amount of strength loss of up to almost 40% occurred. Strength degradation seemed to be less severe when cyclic load at the high stress level was preceded, where apparently different damage modes are expected. This is also corroborated by the fewer premature failures under the high stress level. In fact, a number of 4 tests cycled at the high stress level exhibited residual strength considerably higher than the static strength. 3 of them came from plate 113 which was considered suspect. The fourth was from plate 49 but still with static strength 20% higher than the average so, it was also considered suspect as well. All the data can be seen in Fig.1. Stresses are normalized by the average static tensile strength value of the OB_UD [20]:

\[ X_T = 793.21 \, MPa \]
Fig.1 Residual strength (RST), static (STT) and premature failures shown along with the S-N curve of the material, at R=0.1

The implementation of the best fitting models to the data is presented in Fig.2. Parameters of the models are shown in Table 1. The degradation seems to be insignificant for the high stress level. With the exception of a single datum, residual strength measurements remain within the static strength scatter.

Fig.2 Model predictions for the Residual Tensile strength of UD at 0º after fatigue at R=0.1
Table 1  RST Model parameters for the of UD at 0º after fatigue at R=0.1

<table>
<thead>
<tr>
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<td>OM</td>
<td>$k_1=0.557$</td>
<td>$k_2=2.417$</td>
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<td>-</td>
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<tr>
<td>Y2</td>
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<td>$c=5.8773$</td>
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<td>REI</td>
<td>$k=2.774$</td>
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4.1.2 R=-1

These test series were performed in CCLRC, and WMC. Test results are shown in Fig.3. Coupons originate mostly from plates 53 to 93, which are closer in production to the ones used for static and fatigue characterization of the material. Possibly due to this, residual strength degradation appears to be more consistent than in the case of R=0.1, with less premature failures confined mainly to high life fractions as expected. A question remains for the too many failures at the high stress level both from CCLRC and WMC from various plate numbers (64-113). At this series of tests the high stress level was shifted from 1000 cycles to 5000 and indeed strength seems to degrade for this stress level. Nevertheless, two tests in that level, coming from plates 72 & 113, show inexplicably high strength.

Model predictions for this case are shown in Fig.4 and the parameters used are presented in Table 2. The predictions of OM and Interaction models seem to follow consistently well the degradation trend.
Fig. 3 Residual strength (RST), static (STT) and premature failures shown along with the S-N curve of the material, at R=-1

Fig. 4 Model predictions for the Residual Tensile strength of UD at 0° after fatigue at R=-1
<table>
<thead>
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<td>$k_2=2.0492$</td>
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<td>$c=1.732$</td>
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<td>REI</td>
<td>$k=3.189$</td>
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4.2 UD at 90°

4.2.1 R=0.1

Tests were performed at UP and VUB. The very low tensile strength on the specific direction of the material induced an uncertainty on the accuracy of the results. Evidence of this additional source of scatter can be seen in Fig.5, where certain coupons show residual strength lower than the maximum stress they were cycled at. Such an unexpected response is probably due to handling of this ‘delicate’ material, high gripping forces, slight bending due to grip misalignment or difference in tab thicknesses etc. Such reasons may have caused the—many indeed—premature failures in this case: Although premature failures at UP were kept very low (1 failure), many such failures occurred at VUB (27 failures) possibly due to a suspected misalignment between the grips. As for the UD material at 0°, strength degradation is negligible for the high stress level, 1st. All test results are shown in Fig.5, while model predictions are presented in Fig.6. Derived values of various model parameters are shown in Table 3. Stresses are normalized by the average static tensile strength value transverse to the fibers, [20]: $Y_T = 55.23 \text{ MPa}$

![Graph of UD 90° R=0.1 RST](image)

Fig.5 Residual strength (RST), static (STT) and premature failures shown along with the S-N curve of the material, at R=0.1
Table 3  RST Model parameters for the of UD at 90º after fatigue at R=0.1

<table>
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<td>Y2</td>
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<td>REI</td>
<td>k=3.418</td>
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</table>

Fig.6  Model predictions for the Residual Tensile strength of UD at 90º after fatigue at R=0.1

4.2.2 R=-1
Residual strength tests after fatigue at R=-1 were performed by UP and CRES. Results are presented in Fig.7. Strength is apparently degrading up to 40% for some coupons at high cycle fatigue but nevertheless, only a rough estimate of the degradation trend can be assumed.
Fig. 7 Residual strength (RST), static (STT) and premature failures shown along with the S-N curve of the material, at R=-1

The models proposed are presented in Fig. 8, their parameter values being listed in Table 4.
Table 4  RST Model parameters for the of UD at 90º after fatigue at R=-1

<table>
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<td>OM</td>
<td>(k_1=0.491)</td>
<td>(k_2=3.214)</td>
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<td>-</td>
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<tr>
<td>Y2</td>
<td>(\omega=17.133)</td>
<td>(K=1.64\times10^{-17})</td>
<td>(b=8.520)</td>
<td>(c=7.795)</td>
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<tr>
<td>REI</td>
<td>(k=5.820)</td>
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</table>

### 4.2.3 R=10

From the tests performed, a high number of premature failures occurred, as seen in Fig.9, most probably due to the combination of a slight deviation between the load cells, though within the calibration limits, of the two different test rigs used for fatigue loading and static tests to failure and of the particularly flat S-N curve. Once the load was corrected according to the calibration of the test rig where the fatigue tests were performed, no more premature failures occurred. Compressive stresses are normalized by the average static compressive strength value transverse to the fibers, [20]: \(Y_c = 166.95\) MPa

To avoid buckling problems for the specific loading ratio, the 1st and 2nd stress levels were altered to those corresponding to nominal lives of 50,000 and 200,000 cycles. The residual tensile strength is shown in Fig.10. Although in a number of tests strength degradation of up to 30% was measured, the degradation trend is partially hidden by the scatter of the residual strength data induced by possible damage to the coupons during the residual strength test gripping, test rig eccentricity etc, as mentioned before. It is interesting to note however, that residual strength remains almost within the scatter of the static strength for the case of the million cycles stress level, where degradation would be expected to be more prominent. This fact indicates that probably the ‘sudden death’ behavior would be the appropriate one for this stress ratio. Considering that, it is worthwhile noting that compressive cyclic loads do not seem to have at least any considerable effect on residual tensile strength transverse to the fiber direction of the UD material. The curves shown in Fig.11 and predicting this sudden death degradation of RST were derived using the REI model for a parameter value of \(k=50\).
Fig. 9 Residual (SLi), static (STC) compressive strength and premature failures for the UD at 90° shown along with the S-N curve at R=10.

Fig. 10 Residual (SLi) and static (STT) tensile strength for the UD at 90°.
Fig. 11 Predictions of sudden death (S.D.) model for the RST of UD at 90° after fatigue at R=10.

4.3 SHEAR

4.3.1 R=0.1

Shear properties (static, fatigue, residual strength) of the UD reference material were defined through tensile tests on symmetric ±45 laminate according to ISO 14129 [15]. All tests were performed at UP [21]-[23]. There is clear evidence of strength degradation since early life of the specimens reaching up to almost 40% at high life fractions. A few premature failures were observed during cyclic loading, confined only to the 80% life fraction. Scatter of test data appears increasing with cycles only at the higher stress level. The experimental results are presented in Fig.12. Implementation of the proposed models is shown in Fig.13, while the parameters used can be seen in Table 5. Stresses are normalized by the average static in-plane shear strength value of the OB_UD material, [21]:

\[ S = 56.63 \text{ MPa} \]
Table 5 In-plane RSS Model parameters after fatigue at R=0.1

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<td>REI</td>
<td>k=3.132</td>
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</tr>
<tr>
<td>OM</td>
<td>k1=0.743</td>
<td>k2=1.903</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Y2</td>
<td>ω=12.805</td>
<td>K=3.32E-22</td>
<td>b=11.130</td>
<td>c=4.265</td>
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</table>

Fig.12 Residual (RSS), static (STS) in-plane shear strength test results and premature failures along with the S-N curve under shear loading at R=0.1
Fig. 13 Model predictions for the Residual Shear Strength of UD after fatigue at $R=0.1$.
5 RESIDUAL COMPRESSIVE STRENGTH

Experimental results in both principal material directions (UD at 0° and 90°) after fatigue at all three stress ratios indicate that the residual compressive static strength remains roughly unchanged. Apparently, the damage modes which develop into the material and cause, in the case of coupons tested for RST, a degradation of tensile strength leave compressive strength unaffected. It is interesting that neither when the specimens fail under compressive failure modes, e.g. R=10, the compressive strength seems to degrade significantly. In that case, one can only assume a steep fall of RSC due to the failure events occurring into the material immediately prior to failure and presenting 'sudden death' behaviour.

Test results are shown in Figs.14 to 18. The large scatter observed for the RSC of the UD 0° at R=0.1 (Fig.14) should probably be attributed to plate variation (42 to 113) used for these tests than to inherent mechanisms that increase variation (e.g. buckling of the coupons), since it is equally dispersed above and below the static strength value. This argument is supported by the lower scatter observed for the tests at R=-1 (Fig.15); coupons were cut from plates 102 to 114.

Having in mind the fact of no degradation of compressive strength for on axis tests, a small number of coupons were tested for RSC transversely to the fibres at 50% of their nominal lives, at the three stress ratios (R=0.1, R=-1 and R=10), to investigate if the same behaviour would be observed. Indeed, the results, as seen in Figs.16-18, suggest no apparent degradation. Consequently no further residual strength tests were performed. The single datum in Fig.16 that shows degradation of almost 50% is considered an outlier that by no means supports the argument of degradation for this case.

For modeling of residual compressive strength the sudden death model is proposed, with a value for k in the order of 50.

Compressive stresses are normalized by the average static compressive strength value along and transverse to the fibers, given respectively by [20]:

\[
X_\text{c} = 542.46 \text{ MPa} \\
Y_\text{c} = 166.95 \text{ MPa}
\]
Fig. 14 Residual compressive strength of the UD at 0° after fatigue at R=0.1

Fig. 15 Residual compressive strength of the UD at 0° after fatigue at R=-1
Fig. 16 Residual compressive strength of the UD at 90° after fatigue at R=0.1

Fig. 17 Residual compressive strength of the UD at 90° after fatigue at R=-1
Fig. 18 Normalized residual compressive strength of the UD at 90° after fatigue at R=10
6 VALIDATION OF PROPOSED MODELS

Models are validated based on their prediction of the residual strength degradation trend as well as on their probabilistic predictions. For the first case the degradation curves of MD coupons are produced and compared with experimental data, while for the second one the cumulative density functions (CDF) at specific stress levels and life fractions are produced. Additionally, based on the derived CDFs, degradation curves at specific reliability levels are compared to experimental data.

6.1 MD DEGRADATION CURVES

UD layers with fibers in the loading direction of a composite laminate are the most essential element of the load carrying skin of rotor blades, since the volume fraction of 0° fibers in an MD laminate defines in large both strength and stiffness. The model proposed for the prediction of degradation on the on-axis direction is using the UD predictions scaled down by a factor equal to the static strength ratio of the 0° direction of the MD and UD laminates:

\[ X_{r}^{MD} = \frac{UTS_{MD}}{UTS_{UD}} X_{r}^{UD} \] (10)

The above assumption is validated using MD residual strength experimental data obtained for R=0.1 and R=-1. The results for the three stress levels of the MD at R=0.1 are shown in Fig.19, while model parameters are presented in Table 6.

<table>
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The predictions of RS tests on MD at R=-1 are shown in Fig.20 along with the experimental data. Model parameters are listed in Table 7. Regarding the MD at R=10, no degradation is observed at RST or RSC, see Fig.21, so no model prediction can be shown apart from the sudden death one which is a trivial case.
Table 7 RST Model parameters for the MD after fatigue at R=-1

<table>
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<td>INT</td>
<td>$x=3.0366$</td>
<td>$y=3.5111$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>REI</td>
<td>$k=1.9519$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>OM</td>
<td>$k_1=0.5647$</td>
<td>$k_2=1.9681$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Y2</td>
<td>$\omega=18.5310$</td>
<td>$K=3.557E-6$</td>
<td>$b=8.912$</td>
<td>$c=3.7289$</td>
</tr>
</tbody>
</table>

Fig.19 Residual strength degradation predictions of MD specimens at R=0.1, based on UD RST models.
Fig. 20 Residual strength degradation predictions of MD specimens at $R = -1$, based on UD RST models.

Fig. 21 Experimental data for residual strength degradation of MD specimens at $R = 10$. Data are normalized by an average compressive strength of 3.0323 kN/mm.
6.2 STATISTICAL PREDICTIONS

The data used for evaluation of the statistical predictions of the models originate from testing of ISO [±45]_{S} coupons. This it is the most consistent data set available since all tests for static strength, fatigue life and residual strength properties were performed at a single Institute (UP) under similar conditions. Also the coupons were cut from similar plates while at least 8 coupons were tested at each stress level/life fraction in order to obtain a more significant statistical sample.

Fig.22 Predictions of the proposed models of the CDF of residual strength at the nine stress level-life fractions pairs of the [±45]_{S} laminate, tested at R=0.1.
The statistical distributions predicted by the 5 models proposed (BR, INT, REI, OM, Y2) for each stress level and life fraction are shown in Fig.22. The CDF predictions shown in this figure are indicative since model parameters are derived using the same data used for verification, causing the predictions to be biased.

In order to have an unbiased validation of the models predictions, a set of 13 specimens were tested for residual strength after 110,000 cycles at the stress level corresponding to 220,000 cycles, at R=0.1. The predictions are shown in Fig.23.

![Fig.23 CDF predictions of models for the 50% nominal life at the stress level corresponding to 220,000 cycles.](image)

Based on the analytical expressions for probability distributions of the proposed models, 90% ‘probability of survival’ or ‘reliability’ degradation curves were predicted. In Figures 24 to 26 these predictions are shown for the three stress levels of the [±45]s laminate along with the experimental data.
Fig. 24 90% probability of survival degradation curves for the $[\pm 45]$$_s$ laminate and stress level of the 5000 cycles

Fig. 25 90% probability of survival degradation curves for the $[\pm 45]$$_s$ laminate and stress level of the 50000 cycles
Fig. 26 90% probability of survival degradation curves for the [±45]s laminate and stress level of the 1000000 cycles.
7 CONCLUSIONS

Different phenomenological, engineering models were fitted to experimental data from residual strength tests on a UD laminate (on-axis and transversely to the fibres) as well as on shear data obtained from tensile tests performed on [±45]_s specimens. From the various models presented in [1] only three show acceptable results. These are: The interaction model (INT), the OM model and the 2nd model of Yang (Y2). The REI model, used from several authors, is also shown in the comparison, even though it tends to overestimate residuals strength during a large initial part of the material's fatigue life. Also the linear model (BR) is included in the proposed models since the linear strength degradation assumption appears to be conservative in almost all cases and close to the 90% probability-of-survival curve as can be seen in Figs 24-26.

For the INT and Y2 models, the procedure suggested in [16], [18] and presented in [1] was used for determining their parameters. For the REI and OM models the equivalent static strength concept was implemented for parameter derivation. Alternatively, direct fitting of these models to experimental data at each stress level can be used. This implementation however, requires more experimental data and leads anyhow to similar results.

The REI as well as the Y2 model fail to predict the initial strength decrease observed on the on-axis directions both at R=0.1 and at R=-1 stress ratios. Furthermore, the predictions of REI model are on the non-conservative side in most cases. On the other hand the Y2 model, apart from its inability to model this initial loss of strength, predicts satisfactorily the subsequent average strength degradation. Its complexity of course is a disadvantage e.g. for use in variable amplitude life predictions, but this can be considered of limited importance.

The OM model, developed during the project, is flexible enough to follow well different degradation trends. Its implementation is simple and requires as limited an experimental data set as the interaction model. Its degradation equation is not a monotonically decreasing one for any combination of the parameters \( k_1 \) and \( k_2 \). This is the reason for which it can lead to false results when implementation is based on misleading residual strength data. It is nevertheless suggested for modelling strength degradation when experimental data indicate a clear behaviour.
An equally flexible model also requiring a reasonable amount of data is the interaction (INT) model. It follows well the degradation of average strength in all cases. Given its two parameters, the degradation equation assumed as well as the fact that it is based on direct fitting, it simply and effectively predicts various degradation behaviours, from wear out with initial loss of strength to sudden death.

Finally, the linear mode (BR), as mentioned before, has been proved to lead to conservative predictions in all cases, especially at higher life fractions, and is definitely proposed as a safe model to use in all cases where residual strength data are not available.

Regarding the statistical predictions of the models, it is difficult to draw definite conclusions on their validity due to the small number of tests performed and especially due to the different conditions, different test rigs and plate to plate variations whose effect has been made apparent throughout the project. Nevertheless one safe conclusion is that scatter does not drastically increase for the fibre dominated laminates, while it seems to increase more for the matrix dominated materials. This can be observed for the case of the ±45 test coupons cut from similar plates and were all tested at UP. Nevertheless, given the data, all the models predict similar scatter for the residual strength data, which is not far from the scatter of static strength tests, except from the Y2 model that predicts better the scatter of the ±45 tests.
8 REFERENCES

15. ISO 14129:1997(E) “Fibre-reinforced plastic composites - Determination of the in-plane shear stress/shear strain response, including the in-plane shear modulus and strength, by the ±45° tension test method”